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Large angle QED processes at e^+e^- colliders at energies below 3 GeV

Andrej B. Arbuzov and Eduard A. Kuraev
Bogoliubov Laboratory of Theoretical Physics, JINR, Dubna, 141980, Russia
E-mail: arbuzov@thsun1.jinr.dubna.su

Gennadi V. Fedotovich
Budker Institute for Nuclear Physics, Prospect Nauki, 11, Novosibirsk, 630090, Russia

Nikolay P. Merenkov
Kharkov Institute of Physics and Technology, Kharkov, 310108, Ukraine

Vladimir D. Rushai
Laboratory of Computing Techniques and Automation, JINR, Dubna, 141980, Russia

Luca Trentadue
Dipartimento di Fisica, Università di Parma and INFN, Gruppo Collegato di Parma, 43100 Parma, Italy

Abstract: QED processes at electron–positron colliders are considerd. We present differential cross–sections for large–angle Bhabha scattering, annihilation into muons and photons. Radiative corrections in the first order are taken into account exactly. Leading logarithmic contributions are calculated in all orders by means of the structure–function method. An accuracy of the calculation can be estimated about 0.2%.

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Contents

1 Introduction

At the existing VEPP–2M e^+e^- collider and planned meson factories QED processes of the lowest order of the perturbation theory (PT) play an important role. These processes are to be considered as an essential background when extracting subtle mesons properties from experimental data. QED processes are used also for calibration and monitoring purposes. For instance, large–angle Bhabha scattering is used for a precise determination of luminosity at e^+e^- colliders. That is the reason, why radiative corrections (RC) to QED processes are to be considered in detail.

A lot of attention was paid to the problem of radiative correction calculations to various QED processes, starting more than 50 years ago [1], concerning mainly the lowest order of PT calculations. The accuracy requirements of modern experiments, however, exceed the ones provided by the first order RC. Unfortunately, calculations of radiative corrections in higher orders encounter tremendous technical difficulties. Nevertheless, powerful methods developed in quantum chromodynamics (see paper [2] and references therein) provide a possibility to improve essentially the results obtained earlier. At first we mean the methods based on the renormalization group ideas and on the factorization theorem. Thay allow us to get a differential cross–section of a certain process similar to the Drell–Yan process cross–section, and to consider leading logarithmic RC of higher orders. Meanwhile, nonleading contributions are to be taken from the lowest order PT calculations.

The aim of our paper is to provide relevant theoretical formulae for QED processes, which are required for experiments with CMD–2 and SND detectors at the VEPP–2M (Novosibirsk) collider [3], and at the DAΦNE (Frascati) [4] and BEPC/BES (Beijing) [5] machines. Formulae cited below may be applied also for higher energies (see the Conclusions), if one will take into account weak interaction and higher hadronic resonance contributions.

A cross–section calculated with an account of radiative corrections (RC) in the n-th order of perturbation theory (PT) contains enhanced contributions of the form $(\alpha/\pi)^n L^n$, where $L = \ln(s/m_e^2)$ is the *large* logarithm (for $s \sim 1$ GeV^2, $L \approx 15$). We call these contributions the leading ones. Nonleading contributions have the order $(\alpha/\pi)^n L^m$, $m < n$. The terms, proportional to $(\alpha/\pi)^n L^n$, can be calculated by means of the structure function method [6]. The structure function formalism, based on the renormalization group approach, permits one to keep all leading terms of order $(\alpha L/\pi)^n$, $n = 0, 1, 2\ldots$ explicitly.

In the first order of PT the nonleading terms, proportional to (α/π), can be accounted by means of so–called $\mathcal{K}$–factor, $\mathcal{K} = 1 + (\alpha/\pi)K$. As for nonleading terms of the order $(\alpha/\pi)^2 L$, they can be correctly calculated in the two–loop approximation. We will consider them in further publications [7]. Fortunately, this second order nonleading RC are small: $(\alpha/\pi)^2 L \sim 10^{-4}$. So, the presented formulae guarantee theoretical precision of the order 0.2%. Considering hard photon emission in the first order of PT we distinguish the contributions due to the initial state radiation, the final state radiation, and their interference. The first one always contains large logarithms. The ones due to the final state radiation as well as the initial–final state interference do not enhance the corrections considerably, except the case of Bhabha scattering. In the case of hard photon radiation large logarithms come from kinematical regions, where the photona are emitted along electron and positron beams. We will call this kinematics as the collinear one.

Our work consist in explicit calculations of the Born and the first order RC contributions to differential cross–sections. We apply the structure function method to increase the accuracy.

The paper is organized as follows. In section 2 we consider $\mu^+\mu^-$ production. Both charge–even and charge–odd contributions to the differential cross–section are evaluated. In Sect. 3 we consider the electron–positron scattering process at large angles. In Sect. 4 we investigate electron–positron annihilation into photons. In the Conclusions we discuss the results obtained and estimate the provided precision. Numerical illustrations are given for a realistic experimental set up.

2 Muon Pair Production

Consider the process

$$e^+(p_+) + e^-(p_-) \to \mu^+(q_+) + \mu^-(q_-).$$

Taking into account the photon and Z–boson intermediate states, the differential cross–section in the Born approximation (in the framework of the Standard Model) has the form

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} = \frac{\mathrm{d}\sigma_0}{\mathrm{d}\Omega}\left\{1 + K_W\right\}, \qquad \frac{\mathrm{d}\sigma_0}{\mathrm{d}\Omega} = \frac{\alpha^2\beta}{4s}\left(2 - \beta^2(1 - c^2)\right), \tag{2.1}$$

$$\beta=\sqrt{1-4m_\mu^2/s}, \qquad c=\cos\widehat{\boldsymbol{p}_-\boldsymbol{q}_-}\,, \qquad s=(p_++p_-)^2=4\varepsilon^2,$$

where the centre–of–mass reference system of the initial beams is applied.

Here K_W (we put the explicit expression for it in the Conclusions) represents contributions due to Z–boson intermediate states (see [8] for example), we will neglect them within the accepted precision: $K_W \sim s/M_Z^2 \lesssim 10^{-3}$. We will drop also all other contributions due to weak interactions in higher orders.

Consider first the even, with respect to the $c \leftrightarrow -c$ permutation, part of the one–loop virtual and soft radiative corrections. Using the known in the literature Dirac and Pauli muon form factors and the known soft photon contributions, we obtain

$$\begin{aligned}
\frac{\mathrm{d}\sigma_{\mathrm{even}}^{B+S+V}}{\mathrm{d}\Omega} &= \frac{\mathrm{d}\sigma_0}{\mathrm{d}\Omega}\frac{1}{|1-\Pi(s)|^2}\biggl\{1+\frac{2\alpha}{\pi}\biggl[\biggl[L-2+\frac{1+\beta^2}{2\beta}l_\beta\biggr]\ln\frac{\Delta\varepsilon}{\varepsilon} \\
&\quad +\frac{3}{4}(L-1)+K_{\mathrm{even}}\biggr]\biggr\}, \qquad (2.2)\\
K_{\mathrm{even}} &= \frac{\pi^2}{6}-\frac{5}{4}+\rho\Bigl(\frac{1+\beta^2}{2\beta}-\frac{1}{2}+\frac{1}{4\beta}\Bigr)+\ln\frac{1+\beta}{2}\left(\frac{1}{2\beta}+\frac{1+\beta^2}{\beta}\right) \\
&\quad -\frac{1-\beta^2}{2\beta}\frac{l_\beta}{2-\beta^2(1-c^2)}+\frac{1+\beta^2}{2\beta}\biggl[\frac{\pi^2}{6}+2\mathrm{Li}_2\left(\frac{1-\beta}{1+\beta}\right) \\
&\quad +\rho\ln\frac{1+\beta}{2\beta^2}+2\ln\frac{1+\beta}{2}\ln\frac{1+\beta}{2\beta^2}\biggr],\\
l_\beta &= \ln\frac{1+\beta}{1-\beta}, \quad \rho=\ln\frac{s}{m_\mu^2} \quad L=\ln\frac{s}{m_e^2}, \quad \mathrm{Li}_2(x)\equiv-\int\limits_0^x\frac{\mathrm{d}t}{t}\ln(1-t).
\end{aligned}$$

where $\Delta\varepsilon \ll \varepsilon$ is the maximum energy of soft photons in the centre–of–mass system. $\Pi(s)$ is the vacuum polarization operator, which includes electron, muon, tau–meson and hadron contributions [9].

The odd part of the one–loop correction comes from the interference of Born and box Feynman diagrams and from the interference part of the soft photon emission contribution. It causes the charge asymmetry of the process:

$$\eta=\frac{\mathrm{d}\sigma(c)-\mathrm{d}\sigma(-c)}{\mathrm{d}\sigma(c)+\mathrm{d}\sigma(-c)}\neq 0. \qquad (2.3)$$

The odd part of the differential cross–section has the following form:

$$\frac{\mathrm{d}\sigma_{\mathrm{odd}}^{S+V}}{\mathrm{d}\Omega} = \frac{\mathrm{d}\sigma_0}{\mathrm{d}\Omega}\frac{2\alpha}{\pi}\biggl[2\ln\frac{\Delta\varepsilon}{\varepsilon}\ln\frac{1-\beta c}{1+\beta c}+K_{\mathrm{odd}}\biggr], \qquad (2.4)$$

$$\begin{aligned}
K_{\mathrm{odd}} &= \frac{1}{2}l_-^2-L_-(\rho+l_-)+\mathrm{Li}_2\left(\frac{1-\beta^2}{2(1-\beta c)}\right)+\mathrm{Li}_2\left(\frac{\beta^2(1-c^2)}{1+\beta^2-2\beta c}\right) \\
&\quad -\int\limits_0^{1-\beta^2}\frac{\mathrm{d}x}{x}f(x)\Bigl(1-\frac{x(1+\beta^2-2\beta c)}{(1-\beta c)^2}\Bigr)^{-\frac{1}{2}}+\frac{1}{2-\beta^2(1-c^2)}
\end{aligned}$$

$$
\begin{aligned}
&\times\Big\{-\frac{1-2\beta^2+\beta^2c^2}{1+\beta^2-2\beta c}(\rho+l_-)-\frac{1}{4}(1-\beta^2)\Big[l_-^2-2L_-(l_-+\rho)\\
&+2\mathrm{Li}_2\left(\frac{1-\beta^2}{2(1-\beta c)}\right)\Big]+\beta c\Big[-\frac{\rho}{2\beta^2}+\left(\frac{\pi^2}{12}+\frac{1}{4}\rho^2\right)\left(1-\frac{1}{\beta}-\frac{\beta}{2}+\frac{1}{2\beta^3}\right)\\
&+\frac{1}{\beta}(-1-\frac{\beta^2}{2}+\frac{1}{2\beta^2})\left(\rho\ln\frac{1+\beta}{2}-2\mathrm{Li}_2\left(\frac{1-\beta}{2}\right)-\mathrm{Li}_2\left(-\frac{1-\beta}{1+\beta}\right)\right)\\
&-\frac{1}{2}l_-^2+L_-(\rho+l_-)-\mathrm{Li}_2\left(\frac{1-\beta^2}{2(1-\beta c)}\right)\Big]\Big\}-(c\to -c),
\end{aligned}
$$

$$
\begin{aligned}
f(x) &= \left(\frac{1}{\sqrt{1-x}}-1\right)\ln\frac{\sqrt{x}}{2}-\frac{1}{\sqrt{1-x}}\ln\frac{1+\sqrt{1-x}}{2},\\
l_- &= \ln\frac{1-\beta c}{2}, \qquad L_-=\ln\left(1-\frac{1-\beta^2}{2(1-\beta c)}\right).
\end{aligned}
$$

In the ultra–relativistic limit ($\beta\to 1$) for the even part we obtain:

$$
\begin{aligned}
\left(\frac{\mathrm{d}\sigma_{\mathrm{even}}^{B+S+V}}{\mathrm{d}\Omega}\right)_{\beta\to 1} &= \frac{\mathrm{d}\sigma_0}{\mathrm{d}\Omega}\frac{1}{|1-\Pi(s)|^2}\Big\{1+\frac{2\alpha}{\pi}\Big[(-2+L+\rho)\ln\frac{\Delta\varepsilon}{\varepsilon}\\
&- 2+\frac{3}{4}L+\frac{3}{4}\rho+\frac{\pi^2}{3}\Big]\Big\}. \qquad (2.5)
\end{aligned}
$$

For the odd part in this limit we obtained the same as Khriplovich [10]:

$$
\begin{aligned}
\left(\frac{\mathrm{d}\sigma_{\mathrm{odd}}^{S+V}}{\mathrm{d}\Omega}\right)_{\beta\to 1} &= \frac{\alpha^3}{s\pi}\Big\{2(1+c^2)\Big[\ln(\mathrm{ctg}\frac{\theta}{2})\ln\frac{\varepsilon}{\Delta\varepsilon}+\frac{1}{2}\ln^2(\sin\frac{\theta}{2})-\frac{1}{2}\ln^2(\cos\frac{\theta}{2})\\
&-\frac{1}{4}\mathrm{Li}_2(\sin^2\frac{\theta}{2})+\frac{1}{4}\mathrm{Li}_2(\cos^2\frac{\theta}{2})\Big]+\cos^2\frac{\theta}{2}\ln(\sin\frac{\theta}{2})-\sin^2\frac{\theta}{2}\ln(\cos\frac{\theta}{2})\\
&-\cos\theta\Big[\ln^2(\cos\frac{\theta}{2})+\ln^2(\sin\frac{\theta}{2})\Big]\Big\}. \qquad (2.6)
\end{aligned}
$$

Consider now the process of hard photon emission

$$
e^+(p_+)+e^-(p_-)\to\mu^+(q_+)+\mu^-(q_-)+\gamma(k). \qquad (2.7)
$$

It was studied in detail [11]. The photon energy is assumed to be larger than $\Delta\varepsilon$. The differential cross–section has the form

$$
\begin{aligned}
\mathrm{d}\sigma &= \frac{\alpha^3}{2\pi^2 s^2}R\mathrm{d}\Gamma, \qquad \mathrm{d}\Gamma=\frac{\mathrm{d}^3q_-\mathrm{d}^3q_+\mathrm{d}^3k}{q_-^0q_+^0k^0}\delta^{(4)}(p_++p_--q_--q_+-k), \qquad (2.8)\\
R &= \frac{s}{16(4\pi\alpha)^3}\sum_{spins}|M|^2=R_e+R_\mu+R_{e\mu}\,.
\end{aligned}
$$

The contribution due to the initial state radiation reads

$$
\begin{aligned}
R_e &= A_e+B_e, \qquad (2.9)\\
A_e &= \frac{s}{s_1^2}\Big\{\frac{s}{\chi_+\chi_-}\left(\frac{1}{2}tt_1+\frac{1}{2}uu_1+sm_\mu^2\right)+\frac{1}{\chi_-}(2m_\mu^2\chi_+-u_1\chi_+'-t_1\chi_-')
\end{aligned}
$$

$$
\begin{aligned}
& + \frac{1}{\chi_+}(2m_\mu^2\chi_- - u\chi'_- - t\chi'_+) - \frac{1}{2}\Big(\frac{u_1}{\chi_+} - \frac{t}{\chi_-}\Big)(u-t_1) - \frac{1}{2}\Big(\frac{t_1}{\chi_+} - \frac{u}{\chi_-}\Big)(t-u_1) \\
& - \frac{s}{2\chi_+}\left(2m_\mu^2 + \frac{1}{\chi_-}(2m_\mu^2\chi_+ - u_1\chi'_+ - t_1\chi'_-)\right) \\
& - \frac{s}{2\chi_-}\left(2m_\mu^2 + \frac{1}{\chi_+}(2m_\mu^2\chi_- - u\chi'_- - t\chi'_+)\right)\Bigg\},
\end{aligned}
$$

$$
\begin{aligned}
B_e &= -\frac{s}{s_1^2}\Bigg\{\left[\frac{m_e^2}{\chi_+^2} + \frac{m_e^2}{\chi_-^2}\right]\left(\frac{1}{2}tt_1 + \frac{1}{2}uu_1 + sm_\mu^2\right) - \frac{m_e^2}{\chi_+^2}(2m_\mu^2\chi_- - u\chi'_- - t\chi'_+) \\
& - \frac{m_e^2}{\chi_-^2}(2m_\mu^2\chi_+ - u_1\chi'_+ - t_1\chi'_-)\Bigg\}, \\
s &= 2p_+p_-, \quad s_1 = (q_+ + q_-)^2, \quad t = -2p_-q_-, \quad t_1 = -2p_+q_+, \\
u &= -2p_-q_+, \quad u_1 = -2p_+q_-, \quad \chi_\pm = p_\pm k, \quad \chi'_\pm = q_\pm k.
\end{aligned}
$$

The final state radiation and the interference of the initial and final state radiation contributions are

$$
\begin{aligned}
R_\mu &= A_\mu + B_\mu, \qquad (2.10) \\
A_\mu &= \frac{1}{s}\Bigg\{(tt_1 + uu_1 + 2sm_\mu^2)\frac{q_-q_+}{\chi'_+\chi'_-} - \frac{4m_\mu^2\chi_+\chi_-}{\chi'_+\chi'_-} - \frac{t_1\chi_- + u\chi_+}{\chi'_-} - \frac{u_1\chi_- + t\chi_+}{\chi'_+} \\
& + \left(\frac{t_1}{2\chi'_+} - \frac{u_1}{2\chi'_-}\right)(t-u) + \left(\frac{u}{2\chi'_+} - \frac{t}{2\chi'_-}\right)(u_1 - t_1) \\
& - \frac{q_+q_-}{\chi'_-\chi'_+}\left[(u_1+t_1)\chi_- + (u+t)\chi_+\right]\Bigg\}, \\
B_\mu &= -\frac{tt_1 + uu_1 + 2sm_\mu^2}{2s}\left(\frac{m_\mu^2}{(\chi'_+)^2} + \frac{m_\mu^2}{(\chi'_-)^2}\right) + \frac{1}{s}\Bigg(\frac{m_\mu^2}{(\chi'_-)^2}(t_1\chi_- + u\chi_+) \\
& + \frac{m_\mu^2}{(\chi'_+)^2}(u_1\chi_- + t\chi_+)\Bigg),
\end{aligned}
$$

$$
\begin{aligned}
R_{e\mu} &= -\frac{tt_1 + uu_1 + 2sm_\mu^2}{2s_1}\left(\frac{t}{\chi_-\chi'_-} + \frac{t_1}{\chi_+\chi'_+} - \frac{u}{\chi_-\chi'_+} - \frac{u_1}{\chi_+\chi'_-}\right) \\
& - \frac{2}{s_1}\Bigg\{-t - t_1 + u + u_1 - \frac{1}{2}u_1\Big(\frac{p_-}{\chi_-} - \frac{q_+}{\chi'_+}\Big)(Q\chi'_+ + P\chi_-) \\
& - \frac{1}{2}t_1\Big(\frac{p_-}{\chi_-} - \frac{q_-}{\chi'_-}\Big)(Q\chi'_- - P\chi_-) - m_\mu^2(\chi_+ + \chi_-)(QP) \\
& + \left(\frac{m_\mu^2}{\chi'_+} - \frac{m_\mu^2}{\chi'_-}\right)(\chi_- - \chi_+) - \frac{1}{2}t\Big(\frac{q_+}{\chi'_+} - \frac{p_+}{\chi_+}\Big)(Q\chi'_+ - P\chi_+) \\
& - \frac{1}{2}u\Big(\frac{q_-}{\chi'_-} - \frac{p_+}{\chi_+}\Big)(Q\chi'_- + P\chi_+)\Bigg\}, \qquad (2.11) \\
P &= \frac{p_+}{\chi_+} - \frac{p_-}{\chi_-}, \qquad Q = \frac{q_-}{\chi'_-} - \frac{q_+}{\chi'_+}.
\end{aligned}
$$

Quantity R_e contains collinear and infrared singularities. A_μ and $A_{e\mu}$ have only infrared singularities. B_μ and $B_{e\mu}$ are free from singularities. Quantity R in the ultra–relativistic case is given in Appendix.

After algebraic transformations we get

$$
\begin{aligned}
R_e &= \frac{s}{\chi_-\chi_+}B - \frac{m_e^2}{2\chi_-^2}\,\frac{(t_1^2+u_1^2+2m_\mu^2 s_1)}{s_1^2} - \frac{m_e^2}{2\chi_+^2}\,\frac{(t^2+u^2+2m_\mu^2 s_1)}{s_1^2} + \frac{m_\mu^2}{s_1^2}\Delta_{s_1s_1}\,, \\
R_{e\mu} &= B\Big(\frac{u}{\chi_-\chi'_+} + \frac{u_1}{\chi_+\chi'_-} - \frac{t}{\chi_-\chi'_-} - \frac{t_1}{\chi_+\chi'_+}\Big) + \frac{m_\mu^2}{ss_1}\Delta_{ss_1}\,, \qquad (2.12) \\
R_\mu &= \frac{s_1}{\chi'_-\chi'_+}B + \frac{m_\mu^2}{s^2}\Delta_{ss}\,, \qquad B = \frac{u^2+u_1^2+t^2+t_1^2}{4ss_1}\,, \\
\Delta_{s_1s_1} &= -\frac{(t+u)^2+(t_1+u_1)^2}{2\chi_-\chi_+}\,, \\
\Delta_{ss} &= -\frac{u^2+t_1^2+2sm_\mu^2}{2(\chi'_-)^2} - \frac{u_1^2+t^2+2sm_\mu^2}{2(\chi'_+)^2} + \frac{1}{\chi'_-\chi'_+}(ss_1-s^2+tu+t_1u_1-2sm_\mu^2), \\
\Delta_{ss_1} &= \frac{s+s_1}{2}\Big(\frac{u}{\chi_-\chi'_+} + \frac{u_1}{\chi_+\chi'_-} - \frac{t}{\chi_-\chi'_-} - \frac{t_1}{\chi_+\chi'_+}\Big) + \frac{2(u-t_1)}{\chi'_-} + \frac{2(u_1-t)}{\chi'_+}\,.
\end{aligned}
$$

Note that from these expressions one may in a moment obtain the corresponding matrix element of the cross symmetrical process $e^-\mu^+ \to e^-\mu^+\gamma$. To be rigorous, we have note that in the cross symmetrical channel one has to take into account some additional terms proportional to m_e^2. In our channel they can be shown as follows:

$$
\begin{aligned}
&R_\mu \to R_\mu + \frac{m_e^2}{s^2}\Delta'_{ss}, \\
&\Delta'_{ss} = \frac{2m_\mu^2 s_1}{\chi'_-\chi'_+} + \frac{t+u_1}{\chi'_-} + \frac{t_1+u}{\chi'_+} + \frac{4m_\mu^2}{\chi'_-} + \frac{4m_\mu^2}{\chi'_+}\,.
\end{aligned}
\qquad (2.13)
$$

We checked the matrix element by a comparison with the one used in the `FORTRAN` program [12], describing electron–muon scattering.

The sum of the hard photon contribution, integrated over the photon phase volume with the condition $k^0 > \Delta\varepsilon$, and the contribution due to the soft and virtual photon emission does not depend on the auxiliary parameter $\Delta = \Delta\varepsilon/\varepsilon \ll 1$. The main contribution, proportional to the large logarithm, comes from the integration of R_e in the case of collinear kinematics of photon emission. For definiteness let us consider the case when the photon moves close to the initial electron direction:

$$
\widehat{\boldsymbol{p}_-\boldsymbol{k}} = \theta \le \theta_0 \ll 1, \quad \theta_0 \gg \frac{m_e}{\varepsilon}. \qquad (2.14)
$$

Here we can use

$$
R_e\Big|_{\boldsymbol{k}\parallel\boldsymbol{p}_-} = \frac{s^2}{s_1^2}\left\{\frac{1+(1-x)^2}{x\chi_-} - \frac{m_e^2}{\chi_-^2}(1-x)\right\}\frac{tt_1+uu_1+2sm_\mu^2}{2}, \qquad (2.15)
$$

where x is the energy fraction carried away by the emitted photon, $x = k^0/\varepsilon = 1 - s_1/s$. Performing the integration over the photon emission angles, we can present the corresponding part of the cross–section (a similar contribution of the hard photon emission along the positron is included below also) in the form

$$\left(\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega_-}\right)_{\mathrm{coll}} = C + D, \tag{2.16}$$
$$C = \frac{\alpha}{2\pi}\left(\ln\frac{s}{m_e^2} - 1\right)\int\limits_{\Delta}^{1}\mathrm{d}x\frac{1+(1-x)^2}{x}\left[\frac{\mathrm{d}\tilde{\sigma}_0(1-x,1)}{\mathrm{d}\Omega_-} + \frac{\mathrm{d}\tilde{\sigma}_0(1,1-x)}{\mathrm{d}\Omega_-}\right],$$
$$D = \frac{\alpha}{2\pi}\int\limits_{\Delta}^{1}\mathrm{d}x\left\{x + \frac{1+(1-x)^2}{x}\ln\frac{\theta_0^2}{4}\right\}\left[\frac{\mathrm{d}\tilde{\sigma}_0(1-x,1)}{\mathrm{d}\Omega_-} + \frac{\mathrm{d}\tilde{\sigma}_0(1,1-x)}{\mathrm{d}\Omega_-}\right],$$

where $\mathrm{d}\tilde{\sigma}_0(1-x_1, 1-x_2)/\mathrm{d}\Omega_-$ is the so–called *shifted* Born differential cross–section. It describes the process $e^+(p_+(1-x_2)) + e^-(p_-(1-x_1)) \to \mu^+(q_+) + \mu^-(q_-)$,

$$\frac{\mathrm{d}\tilde{\sigma}_0(z_1,z_2)}{\mathrm{d}\Omega_-} = \frac{\alpha^2}{4s}\,\frac{y_1[z_1^2(Y_1-y_1c)^2 + z_2^2(Y_1+y_1c)^2 + 8z_1z_2m_\mu^2/s]}{z_1^3z_2^3[z_1+z_2-(z_1-z_2)cY_1/y_1]}\,, \tag{2.17}$$
$$y_{1,2}^2 = Y_{1,2}^2 - \frac{4m_\mu^2}{s}, \quad Y_{1,2} = \frac{q^0_{-,+}}{\varepsilon}, \quad z_{1,2} = 1 - x_{1,2}\,.$$

Using the conservation laws

$$z_1 + z_2 = Y_1 + Y_2, \qquad z_1 - z_2 = y_1c_- + y_2c_+, \tag{2.18}$$
$$y_1\sqrt{1-c_-^2} = y_2\sqrt{1-c_+^2}, \quad c_- \equiv c, \quad c_+ = \cos\widehat{\boldsymbol{p}_-\boldsymbol{q}}_+\,,$$

we obtain the energy fraction of the created muon

$$Y_1 = \frac{4m_\mu^2}{s}\,\frac{(z_2-z_1)c}{2z_1z_2 + [4z_1^2z_2^2 - 4(m_\mu^2/s)((z_1+z_2)^2 - (z_1-z_2)^2c^2)]^{1/2}} + \frac{2z_1z_2}{z_1+z_2-c(z_1-z_2)}\,. \tag{2.19}$$

Quantity C, after adding the corrections due to soft and virtual photons, turns out to be the lowest order perturbative expansion of the convolution of the structure function $\mathcal{D}$ with the shifted Born differential cross–section. Quantity D plays role of a compensating term. Namely, in the sum with the contribution of the cross–section due to hard ($k^0 > \Delta\varepsilon$) photon emission at angles (with respect to the electron and positron) larger than θ_0 the dependence on the auxiliary parameters will cancel.

Here we remind about experimental conditions of the final particles detection, mentioned above. They are to be imposed explicitly by introducing the restriction of the following kind:

$$\Theta(z_1,z_2) = \Theta(Y_1 - y_{\mathrm{th}})\Theta(Y_2 - y_{\mathrm{th}})\Theta(\cos^2\Psi_0 - c_+^2)\Theta(\cos^2\Psi_0 - c_-^2), \tag{2.20}$$

where $c_+ = \widehat{\boldsymbol{p}_+\boldsymbol{q}}_+$, $y_{\mathrm{th}}\varepsilon = \varepsilon_{\mathrm{th}}$ is the threshold of the detectors, and the angle Ψ_0 determines the *dead* cones, surrounding beam axes, unattainable for detection. More detailed cuts can be implemented in a Monte Carlo program, using the formulæ given above.

The leading contributions to the cross–section, containing large logarithm L, as may be recognized, combine to the kernel of Altarelli–Parisi–Lipatov evolution equation:

$$\begin{aligned}
\mathrm{d}\sigma &= \int \mathrm{d}z_1 \mathrm{d}z_2 \mathcal{D}(z_1)\mathcal{D}(z_2)\frac{\mathrm{d}\tilde{\sigma}_0(z_1,z_2)}{|1-\Pi(sz_1z_2)|^2}, \qquad (2.21)\\
\mathcal{D}(z) &= \delta(1-z) + \frac{\alpha}{2\pi}(L-1)P^{(1)}(z) + \left(\frac{\alpha}{2\pi}\right)^2 \frac{(L-1)^2}{2!}P^{(2)}(z) + \ldots\ ,\\
P^{(1)}(z) &= \lim_{\Delta\to 0}\left\{\delta(1-z)(2\ln\Delta + \frac{3}{2}) + \Theta(1-z-\Delta)\frac{1+z^2}{1-z}\right\},\\
P^{(2)}(z) &= \int\limits_z^1 \frac{\mathrm{d}t}{t}P^{(1)}(t)P^{(1)}\left(\frac{z}{t}\right), \qquad \int\limits_0^1 \mathrm{d}zP^{(1,2)}(z) = 0.
\end{aligned}$$

This formula is valid in the leading logarithmical approximation. We will modify it by including nonleading contributions and using the smoothed representation for structure functions [6]:

$$\begin{aligned}
\mathcal{D}(z,s) &= \mathcal{D}^{\gamma}(z,s) + \mathcal{D}^{e^+e^-}(z,s), \qquad (2.22)\\
\mathcal{D}^{\gamma}(z,s) &= \frac{1}{2}b\Big(1-z\Big)^{\frac{b}{2}-1}\left[1+\frac{3}{8}b+\frac{b^2}{16}\Big(\frac{9}{8}-\frac{\pi^2}{3}\Big)\right]\\
&\quad -\frac{1}{4}b(1+z) + \frac{1}{32}b^2\Big(4(1+z)\ln\frac{1}{1-z} + \frac{1+3z^2}{1-z}\ln\frac{1}{z} - 5 - z\Big),\\
\mathcal{D}^{e^+e^-}(z,s) &= \frac{1}{2}b\Big(1-z\Big)^{\frac{b}{2}-1}\left[-\frac{b^2}{288}(2L-15)\right]\\
&\quad + \Big(\frac{\alpha}{\pi}\Big)^2\left[\frac{1}{12(1-z)}\Big(1-z-\frac{2m_e}{\varepsilon}\Big)^{\frac{b}{2}}\Big(\ln\frac{s(1-z)^2}{m_e^2}-\frac{5}{3}\Big)^2\right.\\
&\quad \times\Big(1+z^2+\frac{b}{6}\Big(\ln\frac{s(1-z)^2}{m_e^2}-\frac{5}{3}\Big)\Big) + \frac{1}{4}L^2\Big(\frac{2}{3}\,\frac{1-z^3}{z}+\frac{1}{2}(1-z)\\
&\quad \left. +(1+z)\ln z\Big)\right]\Theta(1-z-\frac{2m_e}{\varepsilon}), \qquad b = \frac{2\alpha}{\pi}(L-1).
\end{aligned}$$

In comparison with the corresponding formula in ref. [6] we shifted the terms, arising due to virtual e^+e^- pair production corrections, from $\mathcal{D}^{\gamma}$ into $\mathcal{D}^{e^+e^-}$.

Finally, the differential cross–section can be presented in the form

$$\begin{aligned}
\frac{\mathrm{d}\sigma^{e^+e^-\to\mu^+\mu^-(\gamma)}}{\mathrm{d}\Omega_-} &= \int\limits_{z_{\min}}^1\int\limits_{z_{\min}}^1 \mathrm{d}z_1\mathrm{d}z_2\,\frac{\mathcal{D}(z_1,s)\mathcal{D}(z_2,s)}{|1-\Pi(sz_1z_2)|^2}\,\frac{\mathrm{d}\tilde{\sigma}_0(z_1,z_2)}{\mathrm{d}\Omega_-}\Big(1+\frac{\alpha}{\pi}K\Big)\\
&+\left\{\frac{\alpha^3}{2\pi^2s^2}\int\limits_{\substack{k^0>\Delta\varepsilon\\ \widehat{kp_\pm}>\theta_0}}\frac{R_e|_{m_e=0}}{|1-\Pi(s_1)|^2}\,\frac{\mathrm{d}\Gamma}{\mathrm{d}\Omega_-} + \frac{D}{|1-\Pi(s_1)|^2}\right\}\\
&+\left\{\frac{\alpha^3}{2\pi^2s^2}\int\limits_{k^0>\Delta\varepsilon}\Big(\mathrm{Re}\,\frac{R_{e\mu}}{(1-\Pi(s_1))(1-\Pi(s))^*} + \frac{R_\mu}{|1-\Pi(s)|^2}\Big)\frac{\mathrm{d}\Gamma}{\mathrm{d}\Omega_-}\right.\\
&\left.+\mathrm{Re}\,\frac{C_{e\mu}}{(1-\Pi(s_1))(1-\Pi(s))^*} + \frac{C_\mu}{|1-\Pi(s)|^2}\right\}, \qquad (2.23)
\end{aligned}$$

$$C_\mu = \frac{2\alpha}{\pi}\frac{\mathrm{d}\sigma_0}{\mathrm{d}\Omega_-}\ln\frac{\Delta\varepsilon}{\varepsilon}\left(\frac{1+\beta^2}{2\beta}\ln\frac{1+\beta}{1-\beta}-1\right), \qquad z_{\min} = \frac{2m_\mu}{2\varepsilon - m_\mu},$$
$$C_{e\mu} = \frac{4\alpha}{\pi}\frac{\mathrm{d}\sigma_0}{\mathrm{d}\Omega_-}\ln\frac{\Delta\varepsilon}{\varepsilon}\ \ln\frac{1-\beta c}{1+\beta c}, \qquad K = K_{\mathrm{odd}} + K_{\mathrm{even}},$$

where D, $C_{e\mu}$ and C_μ are compensating terms, which provide cancellation of auxiliary parameters Δ and θ_0 inside figure brackets. In the first term, containing $\mathcal{D}$ functions, we gather all leading terms. A part of nonleading terms proportional to the Born cross–section is written as the $\mathcal{K}$–factor. The rest nonleading terms are written as two additional terms. The compensating term D (see Eq. (2.16)) comes from the integration in the collinear region of hard photon emission. Quantities C_μ and $C_{e\mu}$ come from the even and odd parts of the differential cross–section (arising due to soft and virtual corrections), respectively. Here we consider the phase volumes of two ($\mathrm{d}\Omega_-$) and three ($\mathrm{d}\Gamma$) final particles as the ones, which already include all required experimental cuts. Using the conservation laws Eq. (2.18) and concrete experimental conditions one can define the lower limits of the integration over z_1 and z_2.

There is a peculiar feature in the spectrum of hard photons. Namely, in the end of the spectrum the differential cross–section is proportional to the factor

$$I(s_1) = \frac{2m_\mu^2 + s_1}{s_1^2}\sqrt{1 - \frac{4m_\mu^2}{s_1}}, \tag{2.24}$$

which defines a peak at $s_1 \approx 5.6m_\mu^2$. It comes from the Feynman diagrams describing the emission by the initial particles [13].

3 Large–Angle Bhabha Scattering

The cross–section of Bhabha scattering (corrected by the vacuum polarization factor), which enters into the Drell–Yan form of corrected cross–section, has a bit more complicated form, as far as the scattering and annihilation amplitudes and their interference are to be taken into account. We remind here the form of the Lorentz–invariant matrix element module squared in the Born approximation:

$$\begin{aligned} R_0(s,t,u) &= \frac{1}{16(4\pi\alpha)^4}\sum_{\mathrm{spins}}\left|\mathcal{M}(e^-(p_-)+e^+(p_+)\to e^-(p'_-)+e^+(p'_+))\right|^2 \\ &= \frac{s^2+u^2}{2t^2} + \frac{u^2+t^2}{2s^2} + \frac{u^2}{st}, \qquad\qquad (3.1)\\ s &= (p_-+p_+)^2, \quad t=(p_- - p'_-)^2, \quad u = (p_- - p'_+)^2, \quad s+t+u = \mathcal{O}(m_e^2). \end{aligned}$$

The first term in the right hand side describes the scattering–type Feynman diagram square. The second one corresponds to the square of the annihilation–type diagram. And the third one deals with the interference of the two diagrams. A more compact representation of R_0 is also useful, $R_0 = (1\ +\ s/t\ +\ t/s)^2$. The differential cross–section in the Born approximation has the form

$$\frac{\mathrm{d}\sigma_0^{\mathrm{Born}}}{\mathrm{d}\Omega_-} = \frac{\alpha^2}{4s}\left(\frac{3+c^2}{1-c}\right)^2. \tag{3.2}$$

We will need also quantity R for arbitrary energies of initial particles. Suppose that the initial electron and positron lost a certain energy fraction. The corresponding kinematics is defined as follows:

$$
\begin{aligned}
& e^-(z_1 p_-) \ + \ e^+(z_2 p_+) \ \longrightarrow \ e^-(\tilde{p}_-) \ + \ e^+(\tilde{p}_+), \\
& \tilde{s} = s z_1 z_2, \qquad \tilde{t} = -\frac{1}{2} s z_1 Y_1 (1-c), \qquad \tilde{u} = -\frac{1}{2} s z_2 Y_1 (1+c), \\
& Y_1 = \frac{\tilde{p}^0_-}{\varepsilon} = \frac{2 z_1 z_2}{a}, \qquad a = z_1 + z_2 - (z_1 - z_2) c.
\end{aligned}
$$

Here the *shifted* Born cross–section corrected by vacuum polarization insertions into virtual photon propagators reads

$$
\begin{aligned}
\mathrm{d}\tilde{\sigma}_0(z_1, z_2) &= \frac{4\alpha^2}{s a^2} \biggl\{ \frac{1}{|1-\Pi(\tilde{t})|^2} \, \frac{a^2 + z_2^2 (1+c)^2}{2 z_1^2 (1-c)^2} + \frac{1}{|1-\Pi(\tilde{s})|^2} \, \frac{z_1^2 (1-c)^2 + z_2^2 (1+c)^2}{2a^2} \\
&\quad - \mathrm{Re} \, \frac{1}{(1-\Pi(\tilde{t}))(1-\Pi(\tilde{s}))^*} \, \frac{z_2^2 (1+c)^2}{a z_1 (1-c)} \biggr\} \mathrm{d}\Omega_- \, . \qquad (3.3)
\end{aligned}
$$

Rewriting the known results [14, 15] for the cross–section in the Born approximation with one–loop virtual corrections to it and with the other ones arising due to soft photon emission, we obtain

$$
\begin{aligned}
\frac{\mathrm{d}\sigma_{B+S+V}}{\mathrm{d}\Omega_-} &= \frac{\mathrm{d}\tilde{\sigma}_0(1,1)}{\mathrm{d}\Omega_-} \biggl\{ 1 + \frac{2\alpha}{\pi}(L-1) \left[2 \ln \frac{\Delta\varepsilon}{\varepsilon} + \frac{3}{2} \right] \\
&\quad - \frac{8\alpha}{\pi} \ln(\mathrm{ctg}\frac{\theta}{2}) \ln \frac{\Delta\varepsilon}{\varepsilon} + \frac{\alpha}{\pi} K_{SV} \biggr\}, \qquad (3.4)
\end{aligned}
$$

where

$$
\begin{aligned}
K_{SV} &= -1 - 2\mathrm{Li}_2(\sin^2\frac{\theta}{2}) + 2\mathrm{Li}_2(\cos^2\frac{\theta}{2}) + \frac{1}{(3+c^2)^2} \biggl[\frac{\pi^2}{3}(2c^4 - 3c^3 - 15c) \\
&\quad + 2(2c^4 - 3c^3 + 9c^2 + 3c + 21) \ln^2(\sin\frac{\theta}{2}) - 4(c^4 + c^2 - 2c) \ln^2(\cos\frac{\theta}{2}) \\
&\quad - 4(c^3 + 4c^2 + 5c + 6) \ln^2(\mathrm{tg}\frac{\theta}{2}) + 2(c^3 - 3c^2 + 7c - 5) \ln(\cos\frac{\theta}{2}) \\
&\quad + (\frac{10}{3}c^3 + 10c^2 + 2c + 38) \ln(\sin\frac{\theta}{2}) \biggr] \qquad (3.5)
\end{aligned}
$$

is the part of the $\mathcal{K}$–factor coming from soft and virtual photon corrections,

$$
\begin{aligned}
\frac{\mathrm{d}\tilde{\sigma}_0(1,1)}{\mathrm{d}\Omega_-} &= \frac{\alpha^2}{s} \biggl\{ \frac{5 + 2c + c^2}{2(1-c)^2 |1-\Pi(t)|^2} + \frac{1+c^2}{4|1-\Pi(s)|^2} \\
&\quad - \mathrm{Re} \, \frac{(1+c)^2}{2(1-c)(1-\Pi(t))(1-\Pi(s))^*} \biggr\}, \qquad (3.6) \\
s &= 4\varepsilon^2, \quad t = -s \, \frac{1-c}{2}, \quad u = -s \, \frac{1+c}{2}, \quad c = \cos\theta, \quad \theta = \widehat{\boldsymbol{p}_- \boldsymbol{p}'_-} \, .
\end{aligned}
$$

Quantity $\Delta\varepsilon$ in Eq. (3.4) is the maximum energy of emitted soft photons. $\Pi(s)$ and $\Pi(t)$ are the vacuum polarization operators in the s and t channels. In the Conclusions we will estimate the contribution of weak interactions.

Consider now the process of hard photon (with the energy $\omega = k^0 > \Delta\varepsilon$) emission

$$e^+(p_+) \;+\; e^-(p_-) \;\to\; e^+(p'_+) \;+\; e^-(p'_-) \;+\; \gamma(k).$$

We start with the differential cross–section in the form suggested by F.A. Berends et al. [14] (which is valid for scattering angles being large compared with m_e/ε):

$$\begin{aligned}
\mathrm{d}\sigma_{\mathrm{hard}} &= \frac{\alpha^3}{2\pi^2 s}\,R_{e\bar{e}\gamma}\,\mathrm{d}\Gamma, \qquad \mathrm{d}\Gamma = \frac{\mathrm{d}^3p'_+\mathrm{d}^3p'_-\mathrm{d}^3k}{\varepsilon'_+\varepsilon'_-k^0}\delta^{(4)}(p_+ + p_- - p'_+ - p'_- - k), \qquad (3.7)\\
R_{e\bar{e}\gamma} &= \frac{WT}{4} - \frac{m_e^2}{(\chi'_+)^2}\left(\frac{s}{t}+\frac{t}{s}+1\right)^2 - \frac{m_e^2}{(\chi'_-)^2}\left(\frac{s}{t_1}+\frac{t_1}{s}+1\right)^2 \\
&\quad - \frac{m_e^2}{\chi_+^2}\left(\frac{s_1}{t}+\frac{t}{s_1}+1\right)^2 - \frac{m_e^2}{\chi_-^2}\left(\frac{s_1}{t_1}+\frac{t_1}{s_1}+1\right)^2,
\end{aligned}$$

where

$$\begin{aligned}
W &= \frac{s}{\chi_+\chi_-} + \frac{s_1}{\chi'_+\chi'_-} - \frac{t_1}{\chi'_+\chi_+} - \frac{t}{\chi'_-\chi_-} + \frac{u}{\chi'_+\chi_-} + \frac{u_1}{\chi'_-\chi_+}, \\
T &= \frac{ss_1(s^2+s_1^2) + tt_1(t^2+t_1^2) + uu_1(u^2+u_1^2)}{ss_1tt_1},
\end{aligned}$$

and the invariants are defined as

$$\begin{aligned}
s &= 2p_-p_+, \qquad s_1 = 2p'_-p'_+, \quad t = -2p_-p'_-, \quad t_1 = -2p_+p'_+, \\
u &= -2p_-p'_+, \quad u_1 = -2p_+p'_-, \quad \chi_\pm = kp_\pm, \quad \chi'_\pm = kp'_\pm.
\end{aligned}$$

It is convenient to extract the contribution of the collinear kinematics. We do that for the following reasons. First, it is natural to separate the region with very a sharp behaviour of the cross–section and to consider it carefully. Second, we keep in mind the idea of the leading logarithm factorization, which is valid in all orders of the perturbation theory. We will evaluate the collinear kinematical regions in two different ways. The first one (the quasireal electron approximation) is suitable for a generalization in order to account higher order leading corrections by means of the structure function method. In this way we will obtain below the leading logarithmic contributions and the compensating terms, which will provide the cancellation of auxiliary parameters. The second one (the direct calculation) is more rigorous, it can be used as a check of the first one. We discuss it in detail in preprint [16].

To obtain explicit formulae for compensators it is needed to consider four kinematical regions corresponding to hard photon emission inside narrow cones, surrounding the initial and final charged particle momenta. The vertices of the cones are taken in the interaction point. We introduce a small auxiliary parameter θ_0, it should obey the restriction

$$m_e/\sqrt{s} \ll \theta_0 \ll 1. \qquad (3.8)$$

So, we define a collinear kinematical region, as the part of the whole phase space, in which the hard photon is emitted within the cone of θ_0 polar angle with respect to the direction of motion of one of the charged particles.

Using the method of quasireal electrons [17], the matrix element $\mathcal{M}$ (squared and summed up over polarization states) of the process of hard photon emission can be expressed through a *shifted* matrix element of the process without photon emission (see Eq. (4) in [17]):

$$
\begin{aligned}
\sum|\mathcal{M}(p_1,k,p_1',\mathcal{X})|^2 &= 4\pi\alpha\left[\frac{1+(1-x)^2}{x(1-x)}\,\frac{1}{kp_1}-\frac{m^2}{(kp_1)^2}\right]\sum|\mathcal{M}_0(p_1-k,p_1',\mathcal{X})|^2, \\
\sum|\mathcal{M}(p_1,p_1',k,\mathcal{X})|^2 &= 4\pi\alpha\left[\frac{y^2+Y^2}{\omega Y}\,\frac{\varepsilon}{kp_1'}-\frac{m^2}{(kp_1')^2}\right]\sum|\mathcal{M}_0(p_1,p_1'+k,\mathcal{X})|^2, \quad (3.9) \\
x &= \frac{\omega}{\varepsilon}, \quad p_1^0=\varepsilon, \quad y=\frac{p_1'^{\,0}}{\varepsilon}, \quad Y=x+y,
\end{aligned}
$$

where $\mathcal{X}$ denotes the momenta of non–radiating incoming and outgoing particles in a concrete process. The integration over the phase volume of the emitted photon inside the narrow cone, surrounding its parent charged particle momentum, gives the following factors:

$$
\begin{aligned}
&\frac{4\alpha}{16\pi^2}\int\frac{\mathrm{d}^3k}{\omega}\left[\frac{1+(1-x)^2}{x(1-x)}\,\frac{1}{kp_1}-\frac{m^2}{(kp_1)^2}\right]=\frac{\alpha}{2\pi}\,\frac{\mathrm{d}z_1}{z_1}\Big[P_\Theta^{(1)}(z_1)\Big(L-1+\ln\frac{\theta_0^2}{4}\Big) \\
&\quad +1-z_1\Big], \qquad\qquad z_1=1-x, \qquad\qquad (3.10) \\
&\frac{4\alpha}{16\pi^2}\int\frac{\mathrm{d}^3k}{\omega}\left[\frac{y^2+Y^2}{xY}\,\frac{1}{kp_1'}-\frac{m^2}{(kp_1')^2}\right]=\frac{\alpha}{2\pi}\,\frac{\mathrm{d}z_3}{z_3}\Big[P_\Theta^{(1)}(z_3)\Big(L-1+\ln\frac{\theta_0^2}{4} \\
&\quad +2\ln z_3\Big)+1-z_3\Big], \qquad\qquad z_3=1-\frac{\omega}{p_1'^{\,0}+\omega}=1-\frac{x}{Y}\,.
\end{aligned}
$$

Note that the terms proportional to $(L-1)$ contain the kernel $P^{(1)}$ (see Eq. (2.21)) of Altarelli–Parisi–Lipatov evolution equations (more precisely, they contain Θ–part of the nonsinglet kernel):

$$
P_\Theta^{(1)}(z) = \frac{1+z^2}{1-z}\Theta(1-z-\Delta).
$$

Collecting the contributions of the four collinear regions, we obtain

$$
\begin{aligned}
\frac{\mathrm{d}\sigma_{\mathrm{coll}}}{\mathrm{d}\Omega_-} &= \frac{\alpha}{\pi}\int\limits_\Delta^1\frac{\mathrm{d}x}{x}\left\{\left[\left(1-x+\frac{x^2}{2}\right)\left(L-1+\ln\frac{\theta_0^2}{4}+2\ln(1-x)\right)+\frac{x^2}{2}\right]\right. \\
&\times 2\,\frac{\mathrm{d}\tilde\sigma_0(1,1)}{\mathrm{d}\Omega_-}+\left[\left(1-x+\frac{x^2}{2}\right)\left(L-1+\ln\frac{\theta_0^2}{4}\right)+\frac{x^2}{2}\right] \\
&\times\left.\left[\frac{\mathrm{d}\tilde\sigma_0(1-x,1)}{\mathrm{d}\Omega_-}+\frac{\mathrm{d}\tilde\sigma_0(1,1-x)}{\mathrm{d}\Omega_-}\right]\right\}, \qquad (3.11)
\end{aligned}
$$

where the shifted Born cross–section is defined in Eq. (3.3).

Adding the contributions of virtual and soft photon emission, we restore the complete kernel. Generalizing the procedure for the case of photon emission by all charged particles, we come to the representation of the cross–section in the leading logarithmic approximation. The final expression for the cross–section therefore has the form

$$
\begin{aligned}
\frac{\mathrm{d}\sigma^{e^+e^-\to e^+e^-(\gamma)}}{\mathrm{d}\Omega_-} &= \int\limits_{\bar z_1}^{1}\mathrm{d}z_1\int\limits_{\bar z_2}^{1}\mathrm{d}z_2\,\mathcal{D}(z_1)\mathcal{D}(z_2)\frac{\mathrm{d}\tilde\sigma_0(z_1,z_2)}{\mathrm{d}\Omega_-}\left(1+\frac{\alpha}{\pi}K_{SV}\right)\Theta \\
&\times\int\limits_{y_{\mathrm{th}}}^{Y_1}\frac{\mathrm{d}y_1}{Y_1}\int\limits_{y_{\mathrm{th}}}^{Y_2}\frac{\mathrm{d}y_2}{Y_2}\,\mathcal{D}(\frac{y_1}{Y_1})\mathcal{D}(\frac{y_2}{Y_2}) \\
&+\frac{\alpha}{\pi}\int\limits_{\Delta}^{1}\frac{\mathrm{d}x}{x}\Biggl\{\biggl[\left(1-x+\frac{x^2}{2}\right)\ln\frac{\theta_0^2(1-x)^2}{4}+\frac{x^2}{2}\biggr]\,2\,\frac{\mathrm{d}\sigma_0^{\mathrm{Born}}}{\mathrm{d}\Omega_-} \\
&+\biggl[\left(1-x+\frac{x^2}{2}\right)\ln\frac{\theta_0^2}{4}+\frac{x^2}{2}\biggr]\biggl[\frac{4\alpha^2}{s(1-x)^2[2-x(1-c)]^4} \\
&\times\left(\frac{3-3x+x^2+2x(2-x)c+c^2(1-x+x^2)}{1-c}\right)^2 \\
&+\frac{4\alpha^2}{s[2-x(1+c)]^4}\left(\frac{3-3x+x^2-2x(2-x)c+c^2(1-x+x^2)}{1-c}\right)^2\biggr]\Biggr\}\Theta \\
&-\frac{\alpha^2}{4s}\left(\frac{3+c^2}{1-c}\right)^2\frac{8\alpha}{\pi}\ln(\mathrm{ctg}\frac{\theta}{2})\ln\frac{\Delta\varepsilon}{\varepsilon}\;+\;\frac{\alpha^3}{2\pi^2 s}\int\limits_{\substack{k^0>\Delta\varepsilon\\ \pi-\theta_0>\theta>\theta_0}}\frac{WT}{4}\,\Theta\frac{\mathrm{d}\Gamma}{\mathrm{d}\Omega_-}, \qquad (3.12)
\end{aligned}
$$

$$
Y_1=\frac{2z_1z_2}{z_1+z_2-c(z_1-z_2)}\,,\qquad Y_2=\frac{z_1^2+z_2^2-(z_1^2-z_2^2)c}{z_1+z_2-c(z_1-z_2)}\,,
$$
$$
\bar z_1=\frac{y_{\mathrm{th}}(1+c)}{2-y_{\mathrm{th}}(1-c)}\,,\qquad \bar z_2=\frac{z_1y_{\mathrm{th}}(1-c)}{2z_1-y_{\mathrm{th}}(1+c)}\,.
$$

The last term describes hard photon emission process, provided that the photon energy fraction x is larger than $\Delta=\Delta\varepsilon/\varepsilon$, and its emission angle with respect to any charged particle direction is larger than some small quantity θ_0. The sum of the last 3 terms in Eq. (3.12) does not depend on the auxiliary parameters Δ and θ_0, if they are sufficiently small. We omitted the effects due to vacuum polarization in the last three terms which describe real hard photon emission. Because the theoretical uncertainty, coming from this approximation, has the order $\delta(\mathrm{d}\sigma)/\mathrm{d}\sigma\sim(\alpha/\pi)^2L\lesssim 10^{-4}$. Nevertheless if the centre–off–mass energy is close to some resonance mass (say to m_ϕ) the effect due to vacuum polarization may become visible. The differential cross–section for non–collinear hard photon emission, that takes into account vacuum polarization explicitly, is presented in Appendix.

4 Annihilation of e^+e^- into Photons

Considering the RC due to emission of virtual and soft real photons to the cross–section of two quantum annihilation process

$$
e^+(p_+)\;+\;e^-(p_-)\;\to\;\gamma(q_1)\;+\;\gamma(q_2), \qquad (4.1)
$$

we will use the results obtained in papers [18]:

$$\begin{aligned}
d\sigma_{B+S+V} &= d\tilde{\sigma}_0(1,1)\left\{1+\frac{\alpha}{\pi}\left[(L-1)\left(2\ln\frac{\Delta\varepsilon}{\varepsilon}+\frac{3}{2}\right)+K_{SV}\right]\right\}, \qquad (4.2)\\
K_{SV} &= \frac{\pi^2}{3}+\frac{1-c^2}{2(1+c^2)}\left[\left(1+\frac{3}{2}\frac{1+c}{1-c}\right)\ln\frac{1-c}{2}\right.\\
&\left.+\left(1+\frac{1-c}{1+c}+\frac{1}{2}\frac{1+c}{1-c}\right)\ln^2\frac{1-c}{2} \;+\; (c\to -c)\right],\\
d\tilde{\sigma}_0(1,1) &= \frac{\alpha^2(1+c^2)}{s(1-c^2)}d\Omega_1\,, \quad s=(p_++p_-)^2, \quad c=\cos\theta_1, \quad \theta_1=\widehat{\boldsymbol{q}_1\boldsymbol{p}}_-\,.
\end{aligned}$$

We suppose that the two final photons are registered in an experiment and their polar angles with respect to the initial beam directions are not small ($\theta_{1,2}\gg m_e/\varepsilon$).

Consider the three–quantum annihilation process

$$e^+(p_+) \;+\; e^-(p_-) \;\to\; \gamma(q_1) \;+\; \gamma(q_2) \;+\; \gamma(q_3)$$

with the cross–section (see the paper by M.V. Terentjev [18])

$$\begin{aligned}
d\sigma^{e^+e^-\to 3\gamma} &= \frac{\alpha^3}{8\pi^2 s}\,R_{3\gamma}\,d\Gamma\,, \qquad (4.3)\\
R_{3\gamma} &= s\,\frac{\chi_3^2+(\chi_3')^2}{\chi_1\chi_2\chi_1'\chi_2'}-2m_e^2\left[\frac{\chi_1^2+\chi_2^2}{\chi_1\chi_2(\chi_3')^2}+\frac{(\chi_1')^2+(\chi_2')^2}{\chi_1'\chi_2'\chi_3^2}\right]+\text{(cyclic permut.)},\\
d\Gamma &= \frac{d^3q_1\,d^3q_2\,d^3q_3}{q_1^0q_2^0q_3^0}\delta^{(4)}(p_++p_--q_1-q_2-q_3),
\end{aligned}$$

where

$$\chi_i=q_ip_-, \quad \chi_i'=q_ip_+, \quad i=1,2,3\,.$$

The process can be treated as a radiative correction to the two–quantum annihilation.

In the same way as we have done before we will distinguish the contributions of the collinear kinematical region, when extra photons are emitted within narrow cones of the opening angle $2\theta_0\ll 1$ to one of the charged particles, and the semi–collinear ones, when extra photons are emitted outside these cones. The first one can be obtained using the quasireal electron method [17]. It reads:

$$\begin{aligned}
d\sigma_{\text{coll}} &= \frac{\alpha}{\pi}\int\limits_{\Delta}^{1}\frac{dx}{x}\left[(1-x+\frac{x^2}{2})\left(L-1+\ln\frac{\theta_0^2}{4}\right)+\frac{x^2}{2}\right] \qquad (4.4)\\
&\times\left[d\tilde{\sigma}_0(1-x,1)+d\tilde{\sigma}_0(1,1-x)\right],
\end{aligned}$$

where the *shifted* cross–section has the form

$$d\sigma_0(z_1,z_2)=\frac{2\alpha^2}{s}\,\frac{z_1^2(1-c)^2+z_2^2(1+c)^2}{(1-c^2)(z_1+z_2+(z_2-z_1)c)^2}d\Omega_1\,. \qquad (4.5)$$

Again rearranging the separate contributions and applying the structure functions method we obtain the *improved* cross–section

$$\mathrm{d}\sigma^{e^+e^-\to\gamma\gamma(\gamma)} = \int\limits_{\bar{z}_1}^{1} \mathrm{d}z_1\, \mathcal{D}(z_1) \int\limits_{\bar{z}_2}^{1} \mathrm{d}z_2\, \mathcal{D}(z_2) \mathrm{d}\tilde{\sigma}_0(z_1, z_2) \left(1 + \frac{\alpha}{\pi} K_{SV}\right)$$
$$+ \frac{\alpha}{\pi} \int\limits_{\Delta}^{1} \frac{\mathrm{d}x}{x} \left[\left(1 - x + \frac{x^2}{2}\right) \ln\frac{\theta_0^2}{4} + \frac{x^2}{2}\right] \left[\mathrm{d}\tilde{\sigma}_0(1-x, 1) + \mathrm{d}\tilde{\sigma}_0(1, 1-x)\right]$$
$$+ \frac{1}{3} \int\limits_{\substack{z_i\geq\Delta \\ \pi-\theta_0\geq\theta_i\geq\theta_0}} \frac{4\alpha^3}{\pi^2 s^2} \left[\frac{z_3^2(1+c_3^2)}{z_1^2 z_2^2(1-c_1^2)(1-c_2^2)} + (\text{cyclic permutations})\right] \mathrm{d}\Gamma, \qquad (4.6)$$
$$z_i = \frac{q_i^0}{\varepsilon}, \quad c_i = \cos\theta_i, \quad \theta_i = \widehat{\boldsymbol{p}_- \boldsymbol{q}_i}\,,$$

where lower limits $\bar{z}_{1,2}$ are defined in Eq. (3.12). The multiplier $\frac{1}{3}$ in the last term takes into account the identity of the final photons. The sum of the last two terms does not depend on Δ and θ_0. Note that the annihilation process is a pure QED one, hadronic contributions as well as weak interaction effects are far beyond the required accuracy.

5 Conclusions

Thus we had considered the series of processes at electron–positron colliders of moderately high energies. We presented differential cross–sections to be integrated over concrete experimental conditions. The formulae are good as for semi–analytical integration, as well as for the creation of a Monte Carlo event generator [19]. In a separate publication we are going to present analysis of the effects of radiative corrections for the conditions of VEPP–2M (Novosibirsk), DAΦNE (Frascati) and BEPC/BES (Beijing). The idea of our approach was to separate the contributions due to $2 \to 2$ like processes and $2 \to 3$ like ones. The compensating terms allow us to eliminate the dependence on auxiliary parameters in both contributions separately.

Note that all presented formulae are valid only for large–angle processes. Indeed, in the region of very small angles $\theta \sim m_e/\varepsilon$ of final particles with respect to the beam directions there are contributions of double logarithmic approximation [9]. These small angle regions give the main part of the total cross–section. We suppose that this kinematics is rejected by experimental cuts.

$\theta_\pm$	σ_0^{Born}(mb)	$\delta_{\mathrm{VP}}(\%)$	$\delta_{\mathrm{SF}}^{\mathrm{ini}}(\%)$	$\delta_{\mathrm{SF}}^{\mathrm{fin}}(\%)$	$\delta_{\mathrm{K}}(\%)$	$\delta_\gamma(\%)$	σ_{tot}(mb)
$9^\circ < \theta_\pm < 171^\circ$	$3.77 \cdot 10^{-2}$	1.53	0.11	− 0.30	−1.82	1.50	$3.81 \cdot 10^{-2}$
$1^\circ < \theta_\pm < 179^\circ$	3.08	0.80	0.70	−0.17	− 3.5	4.74	3.16

Table 1: The values of Bhabha cross–section and radiative corrections to it.

In Table 1 and Figure 1 we present some results of numerical calculations according to our formulae. We suppose that a process–event implies detecting of two final particles with the polar angles with respect to the beam axes more than some value Ψ_0. The energies of the particles have to exeed some experimental threshold $\varepsilon_{\rm th}$. A cut–off on the acollinearity of the final particle momenta is possible. But we switched off it in the computations.

In Table 1 we give the values of different RC contributions to large–angle Bhabha scattering cross–section. Here we switched off also the cut–off on the final particle energies (we used only kinematical restrictions). The contributions (see Eq. (3.12)) are defined as follows:

$$\sigma_{tot} \equiv \int \frac{\mathrm{d}\sigma^{e^+e^-\to e^+e^-(\gamma)}}{\mathrm{d}\Omega_-}\,\mathrm{d}\Omega_- = \int \frac{\mathrm{d}\sigma_0^{\rm Born}}{\mathrm{d}\Omega_-}\,\mathrm{d}\Omega_-\left[1+\frac{1}{100\%}\Big(\delta_{\rm VP}+\delta_{\rm SF}^{\rm ini}+\delta_{\rm SF}^{\rm fin}+\delta_{\rm K}+\delta_\gamma\Big)\right], \quad (5.1)$$

where $\delta_{\rm VP}$ is due to the vacuum polarization being included into the Born level diagrams; $\delta_{\rm SF}^{\rm ini(fin)}$ is due to the initial (final) state leading logarithmic corrections; $\delta_{\rm K}$ shows the impact of the $\mathcal{K}$–factor; δ_γ describes the contribution of one hard photon emission at large angles. The effect due to the width of ϕ meson is included as a part of vacuum polarization. It is small for the given integrated cross–sections. But for the description of a differential cross–section it is important, especially for large scattering angles (see [20, 21]).

The Figure illustrates the charge–odd part of the differential cross–section for the $e^+e^- \to \mu^+\mu^-$ process (see Eq. (2.23)). The quantity

$$A_{FB} = \frac{\mathrm{d}\sigma_{\rm odd}^{e^+e^-\to\mu^+\mu^-(\gamma)}/\mathrm{d}c}{\mathrm{d}\sigma_0^{e^+e^-\to\mu^+\mu^-(\gamma)}/\mathrm{d}c}\,100\% \quad (5.2)$$

is shown as a function of c. The short–dashed line represents the contribution due to soft photon emission and virtual corrections. The long–dashed line represents the corresponding odd contribution due to hard photon emission. It comes from the intereference of the amplitudes due to initial and final radiation ($R_{e\mu}$). In the sum of the two contributions the dependence on the auxiliary parameter Δ disappears (it was chosen $\Delta = 0.01$). And we obtain an experimentally measurable asymmetry A_{FB} (solid line). There is also a contribution to the asymmetry due to electroweak interactions. Namely, due to the interference of the Born level amplitudes with γ and Z boson in the s–channel. It can be found from the weak $\mathcal{K}$–factor (5.4). It is included in the total sum (solid line). But for the chosen energies it is really small: it gives a maximal shift of about 0.01% for $E_{\rm beam} = 0.51$ GeV and about 0.1% for $E_{\rm beam} = 1.55$ GeV.

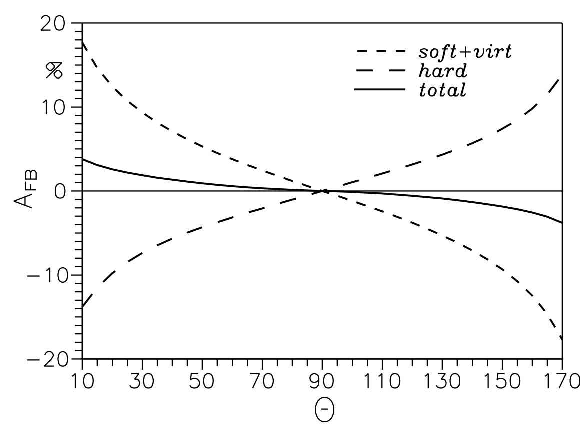

Figure 1: The differential forward–backward asymmetry for $e^+e^- \to \mu^+\mu^-$. The parameters are $\varepsilon = 1.55$ GeV, $\Psi_0 = 10°$, $\varepsilon_{\rm th} = 0.2\varepsilon$.

Let us discuss the accuracy, provided by our formulae. The contribution of weak interaction to the cross–section of muon pair production and Bhabha scattering was parameterized by so–called *weak* $\mathcal{K}$–factors:

$$\mathcal{K}_W = \frac{(\mathrm{d}\sigma)_{\mathrm{EW}} - (\mathrm{d}\sigma)_{\mathrm{QED}}}{(\mathrm{d}\sigma)_{\mathrm{QED}}}\,, \tag{5.3}$$

where quantities $(\mathrm{d}\sigma)_{\mathrm{EW}}$ and $(\mathrm{d}\sigma)_{\mathrm{QED}}$ are the cross–sections calculated in the Born approximation in the frames of the Standard Model and QED, respectively. The weak $\mathcal{K}$–factors (we used the results of papers [8, 22]) are

$$\begin{aligned}
K_W^{e\bar{e}\to\mu\bar{\mu}} &= \frac{s^2(2-\beta^2(1-c^2))^{-1}}{(s-M_Z^2)^2+M_Z^2\Gamma_Z^2}\biggl\{(2-\beta^2(1-c^2))\left(c_v^2\Bigl(3-2\frac{M_Z^2}{s}\Bigr)+c_a^2\right) \\
&\quad - \frac{1-\beta^2}{2}(c_a^2+c_v^2) + c\beta\left[4\Bigl(1-\frac{M_Z^2}{s}\Bigr)c_a^2 + 8c_a^2c_v^2\right]\biggr\}, \qquad (5.4) \\
c_a &= -\frac{1}{2\sin 2\theta_W}, \qquad c_v = c_a(1-4\sin^2\theta_W), \\
K_W^{e\bar{e}\to e\bar{e}} &= \frac{(1-c)^2}{2(3+c^2)^2}\Bigl[4B_1 + (1-c)^2B_2 + (1+c)^2B_3\Bigr] - 1, \qquad (5.5)
\end{aligned}$$

$$\begin{aligned}
B_1 &= (\frac{s}{t})^2\left|1+(g_v^2-g_a^2)\xi\right|^2, \qquad B_2 = \left|1+(g_v^2-g_a^2)\chi\right|^2, \\
B_3 &= \frac{1}{2}\left|1+\frac{s}{t}+(g_v+g_a)^2(\frac{s}{t}\xi+\chi)\right|^2 + \frac{1}{2}\left|1+\frac{s}{t}+(g_v-g_a)^2(\frac{s}{t}\xi+\chi)\right|^2, \\
\chi &= \frac{\Lambda s}{s-m_z^2+iM_Z\Gamma_Z}, \qquad \xi = \frac{\Lambda t}{t-M_Z^2}, \\
\Lambda &= \frac{G_F M_Z^2}{2\sqrt{2}\pi\alpha} = (\sin 2\theta_W)^{-2}, \quad g_a = -\frac{1}{2}, \quad g_v = -\frac{1}{2}(1-4\sin^2\theta_W),
\end{aligned}$$

here θ_W is the weak mixing angle.

These quantities are of order 0.1% up to $\sqrt{s} < 3$ GeV. Contribution of weak interactions to the cross–section of the annihilation into photons (which is absent at the Born level) can be estimated as

$$(\mathcal{K}_W)_{e\bar{e}\to\gamma\gamma} \lesssim \frac{\alpha s}{\pi M_W^2}\,. \tag{5.6}$$

It comes from one–loop electroweak radiative corrections. Another source of uncertainties comes from the approximation of collinear kinematics (or the approximation of quasireal electrons [17]) It can be estimated by the largest omitted terms

$$\frac{\alpha}{\pi}\theta_0^2 \quad \text{and} \quad \frac{\alpha}{\pi}\left(\frac{m_e}{\varepsilon\theta_0}\right)^2. \tag{5.7}$$

Really in the calculations we used the value of the parameter θ_0 of the order 10^{-2} because of the restrictions $\theta_0 \ll 1$ and $\varepsilon\theta_0/m_e \gg 1$. Note that the coefficient before terms of that sort (calculable in principle) is the function of energy fractions and angles, they are of order of 1. For typical values of energy $\varepsilon = 0.5$ GeV this uncertainty is of order $2 \cdot 10^{-4}$ or less.

The third source of uncertainties is the error in the definition of the hadronic vacuum polarization. It has been estimated [9] to be of order 0.04%. For ϕ–meson factories a systematic error in the definition of the ϕ–meson contribution into vacuum polarization is to be added.

Next point concerns nonleading terms of order $(\alpha/\pi)^2 L$. There are several sources of them. One is the emission of two extra hard particles (for the case of Bhabha scattering it was considered in the series of papers [14]). Other are related to virtual and soft–photon radiative corrections to single hard photon emission and Born processes. The most part of these contributions was not considered up to now. Nevertheless, we can estimate the coefficient before the quantity $(\alpha/\pi)^2 L \approx 1 \cdot 10^{-4}$ to be of order of unity. That was indirectly confirmed by our complete calculations of these terms for the case of small–angle Bhabha scattering [23].

Considering all mentioned above sources of uncertainties as independent, we conclude that the systematic error of our formulae does not exceed 0.2%. The main error is due to unknown second–order next–to–leading radiative corrections.

For precise luminosity measurements we suggest to use the large–angle Bhabha scattering process. It has a very large cross–section, a good signature in detectors, and the lowest theoretical uncertainty.

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Appendix

The following expression could be used instead of WT in Eq. (3.12) for more precise definition of the contribution due to non–collinear hard photon emission in large–angle Bhabha scattering:

$$\begin{aligned}(WT)_\Pi &= \frac{(SS)}{|1-\Pi(s)|^2 s\chi'_-\chi'_+} + \frac{(S_1S_1)}{|1-\Pi(s_1)|^2 s_1\chi_-\chi_+} - \frac{(TT)}{|1-\Pi(t)|^2 t\chi_+\chi'_+} \\ &- \frac{(T_1T_1)}{|1-\Pi(t_1)|^2 t_1\chi_-\chi'_-} + \mathrm{Re}\left\{\frac{(TT_1)}{(1-\Pi(t))(1-\Pi(t_1))^* tt_1\chi_-\chi'_-\chi_+\chi'_+}\right. \\ &- \frac{(SS_1)}{(1-\Pi(s))(1-\Pi(s_1))^* ss_1\chi_-\chi'_-\chi_+\chi'_+}\end{aligned} \tag{A.1}$$

$$+\frac{(TS)}{(1-\Pi(t))(1-\Pi(s))^* ts\chi'_-\chi_+\chi'_+}+\frac{(T_1S_1)}{(1-\Pi(t_1))(1-\Pi(s_1))^* t_1s_1\chi_-\chi'_-\chi_+}$$
$$-\frac{(T_1S)}{(1-\Pi(t_1))(1-\Pi(s))^* t_1s\chi_-\chi'_-\chi'_+}-\frac{(TS_1)}{(1-\Pi(t))(1-\Pi(s_1))^* ts_1\chi_-\chi_+\chi'_+}\Bigg\},$$

where

$$\begin{aligned}
(SS) &= (S_1S_1)=t^2+t_1^2+u^2+u_1^2, \qquad (TT)=(T_1T_1)=s^2+s_1^2+u^2+u_1^2,\\
(SS_1) &= (t^2+t_1^2+u^2+u_1^2)(t\chi_+\chi'_+ + t_1\chi_-\chi'_- - u\chi_+\chi'_- - u_1\chi_-\chi'_+),\\
(TT_1) &= (s^2+s_1^2+u^2+u_1^2)(u\chi_+\chi'_- + u_1\chi_-\chi'_+ + s\chi'_-\chi'_+ + s_1\chi_-\chi_+),\\
(TS) &= -\frac{1}{2}(u^2+u_1^2)\left[s(t+s_1)+t(s+t_1)-uu_1\right],\\
(TS_1) &= -\frac{1}{2}(u^2+u_1^2)\left[t(s_1+t_1)+s_1(s+t)-uu_1\right],\\
(T_1S) &= \frac{1}{2}(u^2+u_1^2)\left[t_1(s+t)+s(s_1+t_1)-uu_1\right],\\
(T_1S_1) &= \frac{1}{2}(u^2+u_1^2)\left[s_1(s+t_1)+t_1(s_1+t)-uu_1\right].
\end{aligned}$$

We checked analytically that for the *switched off* vacuum polarization the above formula is equivalent to the multiplication WT in Eq. (3.7):

$$(WT)_\Pi|_{\Pi=0}=WT. \tag{A.2}$$

In the compensating terms, entering into Eq. (3.12), vacuum polarization corrections have to be inserted also. That can be done easily starting with Eq. (3.11). We get

$$\begin{aligned}
\frac{\mathrm{d}\sigma^{e^+e^-\to e^+e^-(\gamma)}_{\mathrm{comp}}}{\mathrm{d}\Omega_-} &= \frac{\alpha}{\pi}\int\limits_{\Delta}^{1}\frac{\mathrm{d}x}{x}\Bigg\{\left[\left(1-x+\frac{x^2}{2}\right)\ln\frac{\theta_0^2(1-x)^2}{4}+\frac{x^2}{2}\right]\\
&\times\frac{\mathrm{d}\tilde{\sigma}_0(1,1)}{\mathrm{d}\Omega_-}\left(1+\frac{1}{(1-x)^2}\right)+\left[\left(1-x+\frac{x^2}{2}\right)\ln\frac{\theta_0^2}{4}+\frac{x^2}{2}\right]\\
&\times\left[\frac{\mathrm{d}\tilde{\sigma}_0(1-x,1)}{\mathrm{d}\Omega_-}+\frac{\mathrm{d}\tilde{\sigma}_0(1,1-x)}{\mathrm{d}\Omega_-}\right]\Bigg\}\\
&-\frac{\mathrm{d}\tilde{\sigma}_0(1,1)}{\mathrm{d}\Omega_-}\frac{8\alpha}{\pi}\ln(\mathrm{ctg}\frac{\theta}{2})\ln\frac{\Delta\varepsilon}{\varepsilon}\,\mathrm{d}\Omega_-\,.
\end{aligned}\tag{A.3}$$

Note that quantity R (see Eq. (2.9)) in the ultra–relativistic limit $s \gg m_\mu^2$ can be derived from Eq. (A.1). One has to omit there all terms except the ones proportional to (SS), (SS_1), (S_1S_1) and divide by 4.

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